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CONTINUITY IN LEIBNIZ'S MATURE METAPHYSICS*

(Received in revised form 20 November 1998)

1. INTRODUCTION

Throughout his philosophical writings, Leibniz attempts to explain fundamental aspects of his metaphysics in terms of continuity. In his early discussions of the structure of matter and motion, he quite explicitly appeals to Aristotle's characterization of continuity, and seems to adopt something like it as his own. There are, of course, interpretative difficulties in discerning what Leibniz takes Aristotle's view to be and how Aristotle himself understands the distinction between a quantity that is continuous and one that is not. But clearly Leibniz employs the Aristotelian distinction, in these early texts, as a way of characterizing the structure of an entity or quantity with respect to its elements or parts: if two things (or parts of a thing) share a boundary, they are continuous.

Commentators usually assume that Leibniz continues to understand the notion of continuity in this way for the rest of his life. I think this interpretation of Leibniz is inadequate. To be sure, he does continue to use something like the Aristotelian conception well into the mature period of his thought. But I believe he articulates a second sense of continuity in his later writings that proves to be of greater importance to the exposition of his mature metaphysics.

To support this interpretation, I shall first lay out a serious interpretative difficulty about the continuity of change, a difficulty that can be resolved only by reinterpreting what Leibniz says about continuity, discontinuity and discreteness in the later metaphysics. I shall then argue that careful consideration of key texts from this period reveals that Leibniz did in fact have at least two notions of continuity in mind, and that once these two senses are distinguished we can reconcile some apparently contradictory texts. In the course

Philosophical Studies **94:** 119–138, 1999.

of this discussion, it will become evident that this distinction has implications not only for Leibniz's treatment of change, but also for other fundamental elements of the mature metaphysics.

2. THE INTERPRETATIVE DIFFICULTY

One of Leibniz's most important metaphysical principles is the Law of Continuity (*lex continuitatis*), according to which "nature never makes leaps" (G V, p. 49). He applies this principle at various ontological levels: at the level of ideal continua like space and time, at the level of phenomena, and at the monadic level. As he states in *The Metaphysical Foundations of Mathematics*, "*continuity* is found in time, extension, qualities, and movement – in fact, in all natural changes, for these never take place by leaps" (after 1714; L, p. 671). Leibniz is remarkably consistent in claiming that ideal continua obey the Law of Continuity. But there are apparent textual inconsistencies in his treatment of actual change. In the passage just cited, Leibniz claims that all natural change is continuous. At the very least, this amounts to the claim that all phenomenal change is continuous. And indeed there are several texts in which Leibniz explicitly endorses such a view. For example, in a 1702 letter to Varignon, Leibniz asks the following:

> Continuity being therefore a necessary prerequisite or a distinctive character of the true laws of the communication of motion, can we doubt that all phenomena are subject to it or become intelligible except by means of the true laws of the communication of motions? (W, p. 186).

Leibniz also says quite explicitly that perceptual change, that is, intra-monadic change, is continuous as well. In the *Monadology*, for example, he says

> I take it as agreed that every created being is subject to change, and therefore, the created monad also, and further that this change is continuous in each one. (1714; L, p. 643)

And in 1704 he writes to DeVolder,

> For me nothing is permanent in things except the law itself which involves a continuous succession and which corresponds, in individual things, to that law which determines the whole world. (L, p. 534)

	Spatial	Temporal
Ideal ⟶	Space	Time; Mathematical Motion
Phenomenal ⟶	Matter or Body	Change in Matter: motion
Most Real ⟶	Monads	Perceptual Change

Figure 1.

So there is good reason to attribute to Leibniz the view that both phenomenal change and intra-monadic or perceptual change are continuous.[1]

Some of Leibniz's letters, however, suggest a quite different view. For example, in a 1705 letter to DeVolder he states that "matter is not continuous but discrete, and actually infinitely divided .... (The same holds of changes, which are not really continuous)" (G II, pp. 278–279). And in a 1705 letter to Princess Sophia he says that "matter appears to be a continuum, but it only appears so, just as does actual motion" (G VII, p. 564). So in the same period that Leibniz claims change is continuous, he also apparently denies the continuity of change.

How are we to interpret these apparently contradictory claims? In what follows I hope to show that we can provide a satisfactory answer to this question only if we are careful to distinguish two senses of continuity in the later metaphysics, and if we are sensitive to what Leibniz means when he claims that something is discrete.

3. ONTOLOGICAL FRAMEWORK

In laying out the interpretative difficulty, I mentioned that Leibniz applies the Law of Continuity to three distinct ontological levels. Let me begin explaining the distinction needed to resolve that difficulty by briefly sketching this three-tiered ontology.[2] In the monadological metaphysics, there are three levels of being and each level has a spatial and a temporal dimension (see Figure 1).

At the ideal level of the spatial dimension is space itself, which is an idealization or an abstraction from phenomena. At the phenomenal level are bodies or matter, which phenomenally 'result from', but are not composed of, an infinite aggregate of monads. Although phenomena are not ultimate constituents of the world, they do have some degree of reality that is derivative from the monads that 'well-found' them. Finally, at the ground-floor metaphysical level are monads, or simple substances. These are the most real things that exist. Each one is completely discernible from any other substance and is fully determinate. Though they get classified in this framework as falling under the spatial dimension, monads are really non-spatial, unextended soul-like beings.

The temporal dimension is to be understood analogously. At the purely ideal level are time and mathematical motion, which are idealizations that are abstracted from the changing states of things. At the phenomenal level is phenomenal change, that is, change, like motion, that occurs in the phenomenal world. Finally, at the most real level are the perceptual states of monads. These states are completely determinate and are discernible from any other state.

There are three important points to keep in mind about this scheme. First, there is a complication at the ground-floor level of the temporal dimension. Clearly, time is something ideal for Leibniz. But it is not clear whether each monad has its own private temporality that metaphysically grounds ideal, public time. If the monad *did* have its own time, the analogy between the spatial dimension and the temporal dimension would break down at the ground-floor level. Monads would be neither in (Newtonian-like) space nor in (Newtonian-like) time; but they would really have temporal properties at the level of deepest metaphysical rigor, even though they would not really have spatial properties.[3] The motivation for this view is that it is hard to understand intra-monadic change (or any kind of change, for that matter) as something that does not take place over time. Notwithstanding this difficulty, I think that for Leibniz the monad *does not* have a sort of private, intra-monadic time, and that the analogy between the spatial and the temporal dimensions remains intact at the level of deepest metaphysical rigor. On this view monads are neither in space nor in time, and at the level of deepest metaphysical rigor, monads do not really have temporal properties, any more than they have spatial properties. So, the temporal ordering of states of a monad (and states of phenomena) is ultimately grounded in states that are not temporally ordered at all.[4]

The second point about this framework is that the division of each tier into two dimensions is somewhat artificial and may even be misleading. For example, it would be wrong to assume that our concept of space can be formed without any experience of phenomenal change,[5] or that the division at the ground-floor metaphysical level is meant to reflect a substantive distinction between a monad and its series of states. Perhaps, then, it would be best to think of the dimensions merely as ways of considering that which exists at each ontological level. For example, at the phenomenal level, we might consider a body at an instant, or we might consider it as it undergoes change. And we might consider space to be an abstraction from phenomena considered in one way, and time an abstraction from phenomena considered in another way.

The third point is that Leibniz believes that the entities at these various levels can be classified as either ideal, on the one hand, or actual or real on the other. Obviously, entities at what we are calling the 'ideal level' are taken by Leibniz to be ideal, while the monads and its states are taken to be real. But what about entities at the phenomenal level? Leibniz fairly consistently classifies bodies and their motions as real or actual, as opposed to ideal. For example, in a letter to DeVolder from 1704 he says that "... in real things, that is, bodies, the parts are not indefinite ..." (L, p. 536).[6] And he is also quite careful to distinguish, depending on the context, between mathematical motion, or motion considered abstractly, and actual motion, or the motion of an actual body.[7] This is not to say that phenomena are as real as monads; Leibniz is clear that the monads are the most genuinely real things that exist. Indeed, he sometimes refers to phenomena as 'semi-mental entities', making the point that phenomena are, in some sense, partially ideal. But phenomena nonetheless have reality because they are well-founded in some set of monads. Their reality is, we might say, derivative from the monads that well-found them.

4. PRELIMINARY REMARKS ABOUT DISCRETENESS

As we have seen, Leibniz says that matter and change are discrete. Some commentators have taken this to mean, or at least imply, that matter comprises discrete bits or 'chunks' and similarly that phenomenal change takes place in discrete slices.[8] I think it is clear this is not what Leibniz means. Let us look more closely at his claim that matter is discrete. In some places, he says that it is discrete insofar as it has a foundation in genuine unities or simples (e.g., L, p. 539). But there are no chunks of matter that could be called simples, since any chunk is itself composed of an infinite number of other chunks. Mathematical points are simple in the sense that they are in no way divisible, but Leibniz is not referring to points when he speaks of units or simples in these passages. He is trying to provide an escape from the labyrinth of the continuum, and his claim is that consideration of the relation between these units and the continuum of matter will provide just such an escape. But he is also clear that thinking of the continuum as composed of unextended, mathematical points is, at least in part, what leads to the labyrinth in the first place.[9]

The claim that matter is discrete should instead be understood as the claim that the simple substances that underlie matter are discrete. As he writes to DeVolder:

> From the things I have said it is also obvious that in actual bodies there is only a discrete quantity, that is, a multitude of monads or of simple substances .... (L, p. 539)

But the fact that matter is well-founded in discrete monads does not imply that there are gaps in matter, or that matter is a composite of discontinuous, though contiguous, parts. To be sure, Leibniz does think that certain facts about the structure of matter follow from its being well-founded in discrete unities; but we cannot infer these facts merely from the claim that matter is discrete. So, it is not yet clear whether there is any contradiction involved in claiming both that matter is discrete and that there are no discontinuities or gaps among its parts.

The issues are a bit more complicated when we consider Leibniz's claim that phenomenal change is discrete. One tempting interpretation is that just as matter is discrete insofar as it results from simple substances that are perfectly unitary and indivisible, so too phenomenal change is discrete insofar as it is founded in monadic states that are perfectly unitary and indivisible. According to this view, although phenomenal change, like matter, may be continuous in structure, it results from discrete, unitary substances whose modes take the form of discrete, unitary perceptual states. As some commentators have put it, in perception we smear the divisions that really exist between perceptual states, making changes appear continuous even though strictly they are not.[10]

Given the close analogy Leibniz draws between matter and phenomenal change, this might seem to be a natural interpretation of Leibniz's comments about the discreteness of change.[11] But it raises difficult questions. Is Leibniz really committed to the existence of these perceptual unities? How could the series of states of a substance be sliced up into perfectly individuated and unified simple states? What could provide the unity for a complex perceptual state? Rather than discuss these difficulties here, I would like to present an alternative interpretation that allows Leibniz both to affirm that change is discrete and to deny that perceptual states are metaphysically simple. So, while I will not directly rule out the possibility that the series of states of a monad is an aggregate of perfectly individuated and unitary states, I do hope to show that Leibniz is not committed to these sorts of discrete states by his claims that change is not continuous.

5. CONTINUITY AND DISCONTINUITY

The idea of continuity that is most often attributed to Leibniz has its roots in Aristotle's *Physics*, and concerns the structure of a quantity with respect to its parts. Leibniz characterizes this sort of continuity, i.e., structural continuity, in various ways, but two are central to his thought. First, he characterizes it in terms of what *we* might call density or compactness, the property a set or quantity has if between any two elements of the set or quantity, there is at least one element interposed between them. In a letter to DeBosses he writes: "…if points are such that there are not two without an intermediate, then a continuous extension is given" (1716; G II, p. 515). Here, Leibniz is clear that he thinks density is at least sufficient for continuity.

It is important to keep in mind that Leibniz's settled view is that points are not fundamental constituents of extended things. Points, as well as lines, surfaces and instants are not parts of things, but are rather *extrema*, *termini* or limits of things (NE, p. 152). An early (1676) text illustrates this point nicely:

[T]here are no points before they are designated. If a sphere touches a plane, the locus of contact is a point; if a body is intersected by another body, or a surface by another surface, then the locus of intersection is a surface or a line respectively. But there are no points, lines or surfaces anywhere else, and there are no extrema in the universe except those that are made by a division. (A VI, iii, pp. 552–553)[12]

Points, instants, etc. are merely modes of mental and semi-mental entities. But more importantly, they do not exist unless they are actually designated.[13] So although it might sound in the DesBosses letter as if Leibniz thinks that points are discrete, fundamental constituents of extension, and although this might sound perfectly natural to us, he does not think that extension, either mathematical or actual, is an aggregate of points. If anything, the characterization in the DesBosses letter is meant to express the view that between any two designated *termini* of a continuous magnitude, it is always possible to designate a further cut or *termini*. And this is just to say that a continuous quantity is infinitely divisible.

The second way that Leibniz characterizes structural continuity is in terms similar to those employed by Aristotle in the *Physics*. In the famous definition at the beginning of Book VI of the *Physics*, continuity gets contrasted with contiguity and successiveness:

Now if the terms 'continuous', 'in contact', and 'in succession' are understood as defined above – things being continuous if their extremities one, in contact if their extremities are together, and in succession if there is nothing of their own kind intermediate between them – nothing that is continuous can be composed of indivisibles .... (*Physics*, VI 1, p. 231a)[14]

Leibniz endorses something like this definition in his early writings on motion and matter. In a 1670 discussion of the coherence of body, for example, he suggests that his view is in accordance with Aristotle's when he notes that "things whose extrema are one ... are continuous or cohering, by Aristotle's definition *also* [i.e., as they are by my definition] ..." (emphasis mine; A VI, 2, p. 266).[15] Unfortunately, there are deep interpretative difficulties concerning how Aristotle's definition is to be understood. In particular, it is difficult to know what the distinction between the continuous and the contiguous is really supposed to amount to. But the central idea behind the definition of 'continuity' is clear enough: the parts of a quantity are continuous if their boundaries or limits are one and the same.

In these early texts, Leibniz most often contrasts continuity with discontinuity, rather than with contiguity or successiveness, though the notion of contiguity is important in these works. He characterizes contiguity in the following way: "*Contiguous* things are those between which there is no distance" (A VI, iii, p. 94). The idea here seems to be that contiguous things are those whose boundaries are distinct, even though the distance between the things is zero, since their boundaries are in contact.[16] Assuming this is the correct interpretation, we can infer the relation between the notions of contiguity and discontinuity from a 1669 letter to Thomasius:

> For by the very fact that the parts are discontinuous, each will have its own separate boundaries [terminos] (for Aristotle defines continuous things as *hon ta eschata hen* [i.e. those whose boundaries are one]). (A VI, ii, p. 435)[17]

Contiguous things are those whose boundaries are not one but rather two; and the idea in the passage just quoted seems to be that discontinuous things, as opposed to continuous things, are those whose boundaries are not one but two. So for Leibniz, the contiguous is to be understood as a species of the discontinuous, where things are discontinuous if their limits are not one.[18,19]

We are now in possession of two independent ways of characterizing the distinction between structural continuity and structural discontinuity. According to the first way:

(1) A series or quantity is continuous if it is dense; a series or quantity is discontinuous if it is not dense.

And according to the second way:

(2) Things are continuous if their limits are one; things are discontinuous if their limits are not one.

It is difficult to say what Leibniz takes the relationship between these two characterizations to be, since different entailment relations among the definitions are possible depending upon how we define the sets or quantities under consideration. For now I am going to leave this question open and simply assume that Leibniz takes (1) and (2) to be different ways of characterizing the same distinction. That is, I will be assuming that a dense quantity just is a quantity whose parts share boundaries, and vice versa, and that this sort of quantity is to be contrasted with one that is not dense and whose parts do not share a boundary.[20]

Fortunately, nothing I say in this paper will depend on whether I am right about this assumption. Indeed, my concern in this section has not been to provide an exhaustive account of Leibniz's views on structural continuity and structural discontinuity, but rather to establish that Leibniz in fact *has* a sense of continuity that is intended as a way to characterize the structure of a quantity with respect to its parts. It is this sense of continuity and discontinuity that is relevant to questions about whether a quantity has gaps, whether a quantity is composed of parts that are in contact, and so on. And it is this sense of continuity that, I will argue, later becomes subsidiary to a second sense.

Assuming, then, that Leibniz's two characterizations of structural continuity are meant to express the same idea, and that Leibniz's mature view is that density is sufficient for structural continuity, let us introduce the following definitions:

S-Continuity: An entity or quantity is *S*-continuous if it is dense.

Discontinuity: An entity or quantity is discontinuous if is it not *S*-continuous.

Let me note two points about these definitions before we turn to Leibniz's second sense of continuity. First, it is no mistake that the term 'discrete' has not been employed in the discussion of structural continuity. It is my view that, at least in the mature metaphysics, Leibniz contrasts discreteness with a sense of continuity that is different from *S*-continuity. This is not to say that Leibniz never uses the term 'discrete' in a way that is meant to imply structural discontinuity. But I think there is evidence that when he is being careful he means something quite different by the term 'discrete' than he does the by the term 'discontinuous'. As I shall argue, for Leibniz, whether something is discrete is entirely independent of whether it is discontinuous.

The second point is that *S*-continuity is the sort of continuity relevant to his *lex continuitatis*. Leibniz would consider any entity or quantity that is discontinuous to be one in which the Law of Continuity is violated, since it contains gaps or leaps of a sort. For even if a quantity has contiguous parts whose boundaries touch one another, there is still a sense in which it contains gaps since the limits of those parts are two, rather than one, and those limits do not have any further limits interposed between them.[21] This point is important to keep in mind as I shall argue that for Leibniz a quantity can be discrete and nonetheless obey the law of continuity.

6. CONTINUITY AND DISCRETENESS

The second sense of continuity that Leibniz employs in the mature metaphysics has its roots in a distinction between the continuous and the discrete that was common in the 17th century. Consider the following idea, which is expressed in a French dictionary from 1690:

(3) A continuous quantity is the sort of quantity dealt with in geometry, whereas a discrete quantity is the sort dealt with in mathematics.[22]

Leibniz never defines continuity in this way, but the idea expressed here hints at a sense of continuity that Leibniz does have in mind. A discrete quantity, such as the number '2', is composed of units or unities, whereas a continuous quantity, such as a line, is not composed at all, though it is divisible into parts. This sort of distinction shows up often in the later metaphysics, and Leibniz sometimes explicitly links this idea to a distinction between ideal wholes and actual wholes. To DeVolder, for example, he says that

> Actual things are compounded as is a number out of unities, ideals as is a number out of fractions; the parts are actually in the real whole but not in the ideal whole. (L, p. 539)[23]

Claims of this sort have led some commentators to think that Leibniz is committed to some version of physical atomism when he says things such as "[m]atter is not continuous but discrete ...." (to DeVolder, G, II, pp. 278–279). But we need to be very careful in interpreting Leibniz when he speaks of parts and units. As I argued earlier, the units Leibniz has in mind when he says things such as "the unit is prior to the multitude" are the monads: "...it is also obvious that in actual bodies there is only a discrete quantity, *that is*, is a multitude of monads or of simple substances ..." (emphasis mine; to DeVolder, L., p. 539). So, a discrete quantity is one that results from genuine unities, the simple substances. This point, taken in conjunction with Leibniz's claim that anything that results from genuine unities is actual or real (as opposed to ideal), is central to the second sense of continuity, the one Leibniz contrasts with the notion of discreteness:

M-Continuity: A quantity is *M*-continuous (metaphysically continuous) just in case (1) it is *S*-continuous and (2) it is mathematically ideal.

Discreteness: A quantity is discrete just in case it is not mathematically ideal (whether or not it is *S*-continuous).

In order to understand this distinction, we need some account of what it is to be mathematically ideal. Leibniz seems to have the following in mind. A mathematically ideal quantity (i) "pertains to possibles and actuals, insofar as they are possible" (G II, p. 282), and (ii) has parts that are completely indeterminate, undifferentiated, indiscernible and uniform. Leibniz offers an excellent characterization of *M*-continuity in a 1706 letter to DesBosses:

A continuous quantity is something ideal which pertains to possibles as well as actuals, insofar as they are possible. A continuum, that is, involves indeterminate parts, but on the other hand, there is nothing indefinite in actuals .... Meanwhile, the knowledge of the continuous, *that is of possibilities*, contains knowledge of eternal truths that are never violated by actual phenomena .... (emphasis mine; G II, p. 282)

Commentators often take this text as claiming that *only* ideal quantities are *S*-continuous, and so any actual quantity must have a discontinuous structure. But the last sentence makes it clear that this is not what Leibniz has in mind. What is true of an ideal continuum like space, with respect to whether its parts are structurally continuous or discontinuous, is also true of actual phenomena, since the law of continuity is violated neither at the ideal level nor at the level of phenomena. The difference is that while the parts of space are completely indeterminate, the parts of matter, or the 'parts' of the motion of a physical object, are completely determinate and differentiated in virtue of their resulting from the only discrete units that exist in the world, the monads.

Further support for this interpretation is found in a letter to DeVolder in which Leibniz suggests that discreteness is to be understood in terms of substantiality while continuity is to be understood in terms of ideality:

Matter is not continuous but discrete, and actually infinitely divided, though no assignable space is without matter. But space, like time, is something not substantial but ideal .... (G II, pp. 278–279)

This text too could be read as supporting the claim that *only* ideal quantities are continuous. But given the monadological metaphysics, a more plausible interpretation is that Leibniz is claiming that a discrete quantity is a real or substantial quantity, that is, a quantity that is founded in simple substances, whereas an *M*-continuous quantity is one that is not real but ideal. The reason that space and time are not discrete is not that they do not have discontinuous structures; it is that they are not substantial, or founded in genuine unities, and so are not completely determinate.

It is important to note that when Leibniz says that the parts are determinate in an actual quantity he does not mean that the quantity is 'carved up' in one way rather than another, resulting in an aggregate of parts each of which have their own boundaries. If the quantity were divided in this way, and the 'determinate' parts had boundaries that were in contact with boundaries of other parts, the result would be a contiguum, not a continuum, and the law of continuity would be violated. So, the determinateness and non-uniformity that a quantity gets from being well-founded in simple substances must be understood in some other way. Towards the end of the next section, I will explain how determinateness and non-uniformity should be understood if we are to preserve *S*-continuity in nature.

7. INTERPRETATIVE DIFFICULTY REVISITED

A close look at the texts from the mature period reveals that there are in fact two senses of continuity that can be attributed to Leibniz. The first, *S*-continuity, is to be contrasted with discontinuity and is the sense that is relevant to whether there are gaps or leaps in a quantity. A discontinuous quantity has gaps whereas an *S*-continuous quantity does not. The second sense of continuity, *M*-continuity, is to be contrasted with discreteness. To attribute *M*-continuity to some quantity is to say more than that it is dense: it is to say that it has a particular ontological status, *viz.*, that it is mathematically ideal. To attribute discreteness to a quantity, on the other hand, is to say that it has some degree of reality, a reality that can only be derived from simple substance.

This interpretation of Leibniz's views on continuity derives support from the help it provides in resolving the textual difficulty presented in section 2. Recall that the interpretative task is to explain how it is that Leibniz can both affirm and deny that natural change is continuous. Given the distinction he draws between *S*-continuity and *M*-continuity, we are now in a position to provide such an explanation. When Leibniz says that motion and perceptual change are continuous, he means that they are *S*-continuous. So, there are no discontinuities of any kind in motion or in the perceptual series of a monad.[24] Change, in both cases, is every bit as structurally continuous as space or time. But when Leibniz says to DeVolder, for example, that change, like matter, is discrete, he is making a different point. Namely, he is claiming that unlike Cartesian extended substance, matter and change are completely determinate since they are well-founded in completely determinate and discernible substances. So although Leibniz thinks that all actual change is *S*-continuous, he thinks that neither type of change is *M*-continuous, since both involve complete determinateness and discernibility of their 'parts'. He makes this point, at least for phenomenal change, quite explicitly in his reply to Bayle's *Rorarius*:

> ... perfectly uniform change is never found in nature ... because the actual world does not remain in this indifference of possibilities but arises from actual divisions or pluralities whose results are the phenomena .... (L, p. 583)

This lack of uniformity in phenomenal change, in virtue of which it is discrete rather than *M*-continuous, is a consequence of the fact that change issues from the only genuinely discrete entities in the world, entities that are themselves completely determinate and discernible. But again, non-uniformity does not entail discontinuity. Leibniz makes this clear in the very next sentence:

> Yet the actual phenomena of nature are arranged, and must be, in such a way that nothing ever happens which violates the law of continuity, ... or any other of the most exact rules of mathematics.[25]

So, the claim that phenomenal change is non-uniform, as well as the claim that the actual world arises from actual divisions, must be interpreted such that it does not undermine the *S*-continuity of motion. The key to such an interpretation is understanding that when Leibniz claims that the 'parts' of a quantity are non-uniform or determinate, he does not mean that the quantity is an aggregate of more fundamental parts, each of which have their own boundaries or limits. Rather, he simply means that any part *that can be delimited* is completely qualitatively determinate and discernible from any other part that can be delimited. So when Leibniz speaks of the parts of an actual motion, he means something like what he means when he speaks of the parts of space or time. In both cases the boundaries of those parts are completely ideal. This way of understanding non-uniformity and determinacy leaves the ontological status of boundaries of the parts within a quantity open, and so leaves open the possibility that a completely determinate and non-uniform quantity is also *S*-continuous.

Something is discrete if it is real, as opposed to ideal, and this reality is derived from simple substances. So whether we are considering a collection of monads, the motion of a billiard ball, or the billiard ball itself, there is a sense in which the reality of the thing is derived from monads. But this might seem a bit puzzling, given the ontological framework discussed in section 3. There I suggested that matter is founded in monads and phenomenal change is founded in monadic change. So, one might wonder, is phenomenal change discrete because it is founded in monads or because it is founded in monadic change? Strictly speaking, phenomenal change is discrete because it is founded in monadic change, which is fully determinate. But to say that motion results from monadic change just is to say that it results from monads conceived of as continually enduring things. According to the interpretation I am advocating, the discreteness of motion does not entail anything about whether motion is, or results from, an aggregate of discrete states. What is doing the work in the account is the determinacy, discernibility and non-uniformity of the parts, since having these properties are sufficient for a quantity's failing to be mathematically ideal. So, Leibniz applies the term 'discrete' to motion because anything in the created world that is determinate is real, and anything that is real owes its reality to the only genuinely discrete things that exist, the monads.

8. CONCLUSION

Let me conclude by stating how I believe the distinctions that have been drawn in this paper apply at each of the three ontological levels.

Ideal Realm: Space, time and mathematical motion are each both *S*-continuous and *M*-continuous. The law of continuity is never violated with respect to these entities and they are each mathematically ideal. It should be noted, however, that not all ideal entities are *M*-continuous. Leibniz is certainly committed to there being ideal spaces that are not structurally continuous. For example, in a 1702 letter to Bayle he says that "space and time . . . relate not only to what actually is, but also to anything that could be put in its place, *just as numbers* are indifferent to the things which can be enumerated" (emphasis mine; L, p. 583). The number '3' and the series of natural numbers are examples of things that are not *S*-continuous, but are nonetheless ideal. These entitles are not, however, discrete in the sense defined since they are not well-founded in created substances. So there are some ideal entities that are neither *M*-continuous nor discrete in the sense defined.[26]

Phenomenal Realm: Matter and its changes are both *S*-continuous, yet they are discrete. The Law of Continuity is never violated at the phenomenal level and neither matter nor its changes are mathematically ideal, since the parts of matter and the 'parts' of its changes are completely determinate and discernible in virtue their resulting from completely determinate simple substances and their states.[27] Importantly, this is not to say that all actual quantities are divided into discrete parts; rather, it is to claim that any parts of an actual quantity that can be delimited will be completely qualitatively determinate and discernible from one another. This is just as we might expect, given Leibniz's claim that phenomena are 'semi-mental'. Phenomena are real insofar as they are founded in real entities, but they are ideal or mental insofar as we contribute spatial and temporal continuity to a world that is, at the level of deepest metaphysical rigor, a *discretum*.

Monadic Realm: The universe of created monads is certainly discrete, as it is an aggregate of completely determinate and real things. But is it *S*-continuous? The answer has to be 'no', since the actual infinite is not a continuum at all. After all, Leibniz's solution to the labyrinth involves arguing that there really *is* no continuum at the level of deepest metaphysical rigor. There may be, however, at least one sense in which monads are continuous, though not *S*-continuous. Leibniz suggests that the series of monads is qualitatively dense; that is, for any two monads that differ qualitatively from one another, there is another monad that lies between these two with respect to qualitative difference.[28] This would satisfy the density requirement for an *S*-continuous quantity, but it would be a notion of continuity that would be difficult to understand in terms of the Aristotelian distinction between a quantity whose parts share a boundary and one whose parts do not. Intra-monadic change is also certainly discrete as it is completely determinate. And on the interpretation that I am advocating there is *S*-continuity in the unfolding of monadic states.

NOTES

* I am indebted to Janet Broughton, Hannah Ginsborg, Peter Hanks, Nicholas Jolley, Alan Nelson, Larry Nolan, and Donald Rutherford for commenting on earlier drafts of this paper. I have also benefited greatly from discussions with Samuel Levey when this paper was delivered at the 1998 annual meeting of the Pacific APA, and from criticisms I received when a version of this paper was presented at the 1997 California Conference in Early Modern Philosophy.

References to Leibniz's work are abbreviated as follows: **A** = *Samtliche Shriften und Briefe. Philosophische Schriften*, (Berlin: Akademie-Verlag, 1923). **AG** = *G.W. Leibniz: Philosophical Essays*, ed. and trans. R. Ariew and D. Garber (Indianapolis: Hackett, 1989). **G** = *Die philosophischen Schriften von Gottfried Wilhem Leibniz*, ed. C.I. Gerhardt, 7 vols. (Berlin: Weidmannsche Buchhandlung, 1875–90). **GM** = *G.W. Leibniz: Mathematische Schriften*, ed. C.I. Gerhardt, 7 vols. (Berlin: A. Asher; Halle: H.W. Schmidt, 1849–63). **L** = *Gottfried Wilhelm Leibniz: Philosophical Papers and Letters*, ed. and trans. L.E. Loemker, 2nd ed. (Dordrecht: Reidel, 1969). **LA** = *The Leibniz-Arnauld Correspondence*, ed. and trans. H.T. Mason (Manchester: Manchester U. Press, 1967; New York: Garland, 1985). **NE** = G.W. Leibniz, *New Essays on Human Understanding*, trans. and ed. P. Remnant and J. Bennett (Cambridge: Cambridge University Press, 1981, 2d ed. 1996) – pagination follows that of A VI, 6. **W** = *Leibniz Selections*, ed. P.P. Wiener (New York: Charles Scribner's Sons, 1951). Translations are mine unless otherwise noted.

[1] There may be some difficulty in drawing the distinction between phenomenal change and perceptual change at all. The distinction is important to note, however, since it is drawn by other commentators who have considered these issues. In this paper, I will be using the term 'phenomenal change' to refer to the sort of change that bodies undergo.
[2] This conception of Leibniz's ontology has become standard. In large part this is due to two interesting and influential papers, the first by J.E. McGuire (1976), and the second by Glenn Hartz and Jan Cover (1988).
[3] If I understand them correctly, this interpretation is advocated by Russell (1900, pp. 127–130), and Rescher (1979, p. 88).
[4] Since this issue is peripheral to my discussion, I will not argue for my interpretation here.
[5] In his Fifth Paper to Clarke (L, p. 703), Leibniz is clear that our observation of the change of place of a body, relative to surrounding bodies that do not change their relation to one another, is necessary for the formation of our idea of space.
[6] See also L, p. 539; and LA, p. 120.
[7] See e.g. G VII, p. 564.
[8] See McGuire (1976), and Hartz and Cover (1988).
[9] See L, p. 456; and LA, p. 120.
[10] Hartz and Cover (1988, p. 508).
[11] For evidence that Leibniz thinks matter and motion are to be understood analogously, see previously noted texts, G II, pp. 278–279; and G VII, p. 564; see also L, pp. 535–536.
[12] Translated by Samuel Levey (1999, p. 63).
[13] There are interesting questions about what counts as a designation when we are considering an actual (or, phenomenal) quantity. It could be that a designation occurs with any division in, say, matter, independently of a designator. Or, it could be that designations are conceptual and made only by beings capable of designating. My view is that in a phenomenal quantity, designation of *termini* of the parts are only ideal, though I am not going to try to defend that view in this paper.
[14] From *The Complete Works of Aristotle*, Revised Oxford Translation, ed. Jonathan Barnes (Princeton University Press, 1984) pp. 390–391.
[15] See also G VII, p. 573.
[16] For a nice text illustrating this point, see A VI, ii, pp. 435–436.
[17] Sam Levey brought this, and the immediately preceding text to my attention in his (1998). Both translations are his.
[18] Leibniz offers a more refined definition of 'continuity', which seems to capture the idea he takes over from Aristotle, in the *Specimen Geometriae Luciferae*: "A continuous whole is one such that its co-integrating parts (i.e., its parts that taken together coincide with the whole) have something in common, and moreover such that if its parts are not redundant (i.e., they have no part in common, that is their aggregate is equal to the whole) they have at least a limit [terminum] in common" (GM VII, p. 284; see also GM V, p. 184).

[19] Just as it is difficult to discern how this distinction between continuity and contiguity is understood by Aristotle, it is similarly difficult to discern how this distinction is understood by Leibniz. I have claimed in this paper that for Leibniz the contiguous is a species of the discontinuous; but there are texts which might cause problems for this reading. For example, in a 1670 letter to Hobbes, Leibniz says that "Bodies whose boundaries are one ... are according to Aristotle's definition *not only contiguous but continuous*, and truly one body ..." (emphasis mine; G VII, p. 573). This suggests that things could be both continuous and contiguous at the same time.

[20] This needs qualification. The assumption I wish to make here is restricted to a domain of discourse about *continua*. Leibniz sometimes speaks of the continuity of species or forms. In such contexts, understanding continuity in terms of density might make some sense, whereas it is not clear that understanding continuity in terms of the sameness of boundaries would.

[21] It is difficult to make this any more precise without running the risk of anachronism. For an excellent discussion of how we might formulate Leibniz's views about structural continuity using point-set topology, see Levey (1998). For an interesting discussion of the relation between Leibnizian structural continuity and combinatorial topology, see Arthur (1986).

[22] "On distingue en Philosophie la quantité continuë de la quantité discrette. La continuë est celle lignes, des superficies & des solides, que est l'objet de la Geometrie. La discrette est celle des nombres, qui est l'objet de l'Arithmetique" (*Le Dictionnaire Universal*, SNL-Le Robert (Paris, 1978)).

[23] See also G II, pp. 278–279, 336, and 379.

[24] I acknowledge that there are some texts that pose a problem for this interpretation. Most of these texts, however, seem to be expressions of an extremely occasionalistic view of change. For example, Hartz and Cover (1988, pp. 500–501) discuss a letter to Princess Sophia in which Leibniz says that actual motion consists of a mass of momentaneous states that are the result of a mass of divine bursts of God ["l'amas d'une infinité d'eclats de la Divinité" (G VII, p. 564)]. Such texts are going to be difficult for any interpreter to reconcile with the many texts in which Leibniz argues against occasionalism.

[25] See also Leibniz's *Letter to Varignon*: "Yet one can say in general that though continuity is something ideal and there is never anything in nature with perfectly uniform parts, the real, in turn, never ceases to be governed perfectly by the ideal and the abstract ..." (L, p. 544).

[26] I am grateful to Donald Rutherford for drawing my attention to some texts that are relevant to this point.

[27] There is a complication that arises for this interpretation with respect to matter that does not arise for phenomenal change. Namely, Leibniz claims that matter is actually infinitely divided. (It is worth noting that in several places, Leibniz makes the weaker claim that matter is infinitely divisible. E.g. see AG, p. 103, and L, p. 544.) On the view I am advocating being infinitely divided is not what it *is* to be discrete, nor does it *follow* from a quantity's being discrete. So we need to either take this claim quite literally, in which case the Law of Continuity is

violated throughout nature, or provide a reinterpretation of Leibniz's claim that a phenomenal quantity is actually infinitely divided. Given that Leibniz sometimes does seem to confuse metaphysical levels in his writings (for example, carelessly referring to monads as 'parts' of bodies), I think there is hope for this kind of interpretative move. However, I shall not attempt to make that sort of move in this paper.

28 Russell discusses, and cites texts relevant to, qualitative continuity among substances in his (1900, pp. 64–65).

REFERENCES

Arthur, R. (1986): 'Leibniz on Continuity', *PSA* 1, 107–115.

Hartz G. and Cover J. (1988): 'Space and Time in the Leibnizian Metaphysic', *Nous* 22, 493–519.

Levy S. (1998): 'Matter and Two Concepts of Continuity in Leibniz', *Philosophical Studies*, this issue.

Levey S. (1999): 'Leibniz on Mathematics and the Actually Infinite Division of Matter', *Philosophical Review* 107(1), 49–96.

McGuire, J.E. (1976): ' 'Labyrinthus continui': Leibniz on Substance, Activity, and Matter', in P. Machamer and R. Turnbull (eds.), *Motion and Time, Space and Matter*, Columbus: Ohio State University Press, pp. 290–326.

Rescher, N. (1979): *Leibniz: An Introduction to His Philosophy*, New Jersey: APQ Library of Philosophy, Rowman and Littlefield.

Russell, B. (1900): *A Critical Exposition of the Philosophy of Leibniz*, Cambridge: Cambridge University Press.

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