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Leibniz on Shape and the Cartesian Conception of Body

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Descartes' philosophy represents an extraordinary attempt to achieve the rationalist goal of acquiring substantive knowledge about the world by employing reason and *a priori* methods. According to Descartes, human beings have the capacity to acquire knowledge of a broad range of metaphysical facts using only the innate conceptual resources with which we are born. We can, for example, come to possess knowledge of the nature and existence of God, mind, and the corporeal world, and we can discover the laws that govern change in nature, without any appeal to experience. In fact, even natural science is *a priori* in the sense that it should proceed in the same way that mathematical reasoning does, namely, by deducing truths from axioms and by employing algebraic reasoning.

There are two aspects of Descartes' system that make such a wide breadth of *a priori* knowledge possible. The first is the existence in human beings of a rational faculty that comes well stocked with a variety of ideas that have been implanted in us by an infinitely benevolent, and thus non-deceiving, God. This cache of ideas, and the ability to perceive in a clear and distinct way what is contained within them, allows us to draw inferences about physics and metaphysics that are presumed to be every bit as transparent as those drawn in mathematics or geometry. The second aspect, which is just as important, is the existence of a world of objects and events that is accurately represented by those ideas – a world that can be understood *a priori* in virtue of its having an intrinsically rational structure.

This commitment to intelligibility can be found in Descartes' thinking about a variety of issues, but clearly the centerpiece of his rationalist efforts is his conception of corporeal reality. The objects of the corporeal world, he argues, are every bit as intelligible as the objects studied in geometry and mathematics because bodies are in essence nothing more than geometrical objects. Knowledge of the essence and properties of geometrical figures is, therefore, also knowledge of the essence and properties of bodies; and geometrical reasoning, which has long been considered to be the paradigm of intelligibility, is nothing less than reasoning about the properties of corporeal things. Of course, there is an important difference between the realm of geometrical space and the created corporeal world: there is motion in the corporeal world. In fact, Cartesian bodies are necessarily in motion relative to one another because motion is responsible for individuating one part of spatial extension from another; what it is to be one body, as distinct from others, is to be a parcel of matter whose parts are moving in concert. Thus, in addition to the purely geometrical properties of size and shape, bodies also have the property of being in motion relative to other bodies. Accordingly, physics and geometry differ insofar as geometry is concerned only with resting forms of space whereas physics is concerned with forms of space in motion relative to one another. But, for Descartes, this difference does not imply that physics is any less an *a priori* science than geometry. The world of bodies in motion can still be exhaustively characterized in quantitative terms. And facts about body can be derived from axioms established *a priori*. The only difference is that the axioms of physics are not derivable from our idea of spatial extension alone but are additionally dependent upon our innate idea of God.

This conception of matter was quite influential and was still widely accepted by philosophers when Leibniz began his own thinking about physics and the nature of body. Early in his career, Leibniz had been charmed by the elegant explanations of natural phenomena offered by the Cartesians, and he continued to countenance such explanations throughout his life. But he soon realized that Descartes' conception of corporeal substance is not sufficient to account for the reality or substantiality of matter. Having stripped body of sensible qualities, forms, forces, strivings, etc., Descartes was left with something that could at best be an abstraction from our phenomenal experiences. To be sure, Leibniz believed, explanations of corporeal phenomena in terms of size, shape, and motion are sufficient in physics because physics is a science that makes extensive use of abstraction. But if bodies are real they must be more than the mere abstractions employed in our thought about them: they must be substantial.

This general criticism brings out something important about efforts to understand corporeal reality in purely quantitative terms: although the complete mathematization of nature would be ideal from the perspective of a rationalist epistemology, there are limitations to mathematization that undermine the possibility that such a characterization could be exhaustive. Mathematics is the science of ideal spaces and entities, such as numbers and geometrical forms. So the more we push in the direction of purely quantitative accounts of nature, the more we move in the direction of abstraction and idealization, and away from the concrete reality we purport to be characterizing. Pushing too hard in the direction of abstraction, then, inevitably results in characterizations of ideal, rather than substantial, entities. Certainly, if we are doing physics it is legitimate to focus solely upon quantitative aspects of body, such as kinematic and geometrical properties. But we run into problems when we succumb to the urge to reduce body to those characteristics. Thus, a tension seems to exist between a desire to maintain a realist conception of body and a rationalist urge to characterize body purely quantitatively. Descartes' mistake, according to Leibniz, was his failure to come to terms with this tension.

In this chapter, I offer a detailed account of one of Leibniz's central objections to the geometrical conception of body. According to this objection, Descartes' theory cannot account for the reality of matter because one of its central explanatory resources, the mode of shape, is imaginary in some way. Leibniz's reasoning about shape is important for two reasons. First, since his arguments purport to show that shapes are at best mere abstractions, they highlight the tension Leibniz sees between realism about body and the mathematization of nature. Second, the arguments play a central role in

Leibniz's early thinking about the possibility of extended substance more generally. As I hope to make clear in this chapter, Leibniz's movement towards an immaterialist metaphysics is at least partially motivated by his sensitivity to the tension between realism and mathematization that Descartes fails to recognize.

The Imaginary Status of Shape: The "Diachronic" Argument

Leibniz first suggests that there is something imaginary about Cartesian extension and its modes in an untitled fragment from the late 1670s (ca. 1678–9):

> That matter and motion are only phenomena, or contain in themselves something imaginary, can be understood from the fact that different and contradictory hypotheses can be made about them, all of which nevertheless satisfy the phenomena perfectly, so that no reason can be devised for determining which of them should be preferred. (A VI, iv, 277; RA 257)

The context of this fragment suggests he is thinking specifically about the relativity of motion, that is, the fact that motion is nothing more than change of position relative to other bodies. His point is that although it may be natural for us, when we see bodies changing position relative to one another, to think that certain bodies have an "absolute motion," this is a mistake because there are other possible frames of reference in which the very same bodies we took to have an absolute motion are at rest. As he says in an essay from 1677:

> The absolute motion we imagine to ourselves, however, is nothing but an affection of our soul while we consider ourselves or other things as immobile, since we are able to understand everything more easily when these things are considered as immobile. (A VI, iv, 360; RA 229)

There are two points here that are analogous to the points he will eventually bring out about shape. The first is a point about metaphysics: there is no absolute motion; there is no reason in the nature of things themselves for saying that one thing is in motion rather than another. The second is a point about the perceiving subject: the apparent (or conceived) absolute motion is merely imagined – it is an affection of the soul, not the world. Although he is not explicit about this, the reason absolute motion must be imaginary seems to be that it does not exist in the nature of things outside of the mind. His justification for the ideality of this quality is thus metaphysical rather than epistemological.

Leibniz makes the analogous metaphysical point about shape in a 1683 essay entitled the "Wonders Concerning the Nature of Corporeal Substance." In this piece, Leibniz argues that although extension and motion are less confused than other qualities, they cannot really be distinctly understood at all. The reason is that

> on the one hand we are always embroiled in the difficulties concerning the composition of the continuum and the infinite, and on the other, because there are in fact no precise shapes in the nature of things, and consequently, no precise motions. (A VI, iv, 279; RA 263)

Unfortunately, Leibniz does not offer any reason for thinking that there are no precise shapes. But the text is worth noting for a couple of reasons. First, Leibniz says explicitly that the mode of shape has a kind of priority over motion, such that showing there are no shapes is sufficient for showing there are no motions. So even if concerns about the relativity of motion can be overcome, there is still a problem for motion that results from the fact that there are no shapes. Second, although it looks as if he is saying something merely about our ability to understand these qualities distinctly, he goes on to make the stronger claim that these modes of extension are phenomena "just as color and sound are phenomena, rather than attributes of things containing a certain absolute nature without relation to us" (A VI, iv, 1465; RA 263). The modes of extension, he suggests, are like secondary qualities in that both are dependent upon the mind in much the same way.

Leibniz begins to present explicit reasons for the claim that there are no determinate shapes in things in the mid-1680s. Many of the arguments look quite similar, and so it is difficult to determine how many arguments there are and how they should be individuated from one another. Some commentators, however, have noted that Leibniz seems to present two distinct lines of reasoning: a "diachronically based" argument and a synchronic argument (Adams 1994: 229–32; Levey forthcoming (a); Sleigh 1990: 211). Roughly, this is correct; one argument explicitly contains diachronic considerations and the other (which I will call the "dominant synchronic argument") does not. However, there are two ways in which contrasting the arguments in terms of a diachronic/synchronic distinction is misleading. First, the argument commentators refer to as diachronic actually contains both diachronic and synchronic considerations. Second, the most interesting difference between the two arguments has to do with the ways in which they purport to demonstrate the impossibility of momentary shapes, rather than with the presence or absence of diachronic considerations. As we will see, the diachronically based argument is quite important to our understanding of Leibniz's thinking in this period because it makes explicit the overall structure of Leibniz's attack on the reality of shape. It is also important, however, because thinking about the strength of the diachronic element of the argument helps us see something important about Leibniz's intentions with respect to the scope of his attack: he intends to show that the reality of shape is inconsistent with any intelligible plenum conception of the world.

Leibniz presents the "diachronically based" argument in only one place, an essay from the mid-1680s entitled "There Is No Perfect Shape in Bodies." The central premise in this argument, as in all the arguments concerning determinate shape, is that matter, which Leibniz believes is a plenum, is actually infinitely divided. These divisions are the result of differing motions of the various parts of matter. So, infinite division of any part of matter implies infinite variety in the motions of its parts. Given this:

> It is true that it will always be possible to draw an imaginary line at each instant; but that line will endure in the same parts only for that instant, because each part has a motion different from every other, since it expresses the whole universe differently. Thus there is no body that has any shape for a definite time, however short it might be. Now I believe that what exists only at a moment has no existence, since it starts and finishes at the same time. (A VI, iv, 1630; RA 297)

We can formulate this argument (hereafter, the NPS argument) in the following way. Assume there is some determinate shape in the plenum (or a body with some determinate shape); either this shape endures for some period of time or it exists for a moment; it cannot exist for some period of time (however short), because every part of the shape has a motion that is different from every other and this change is continuous; but it cannot exist for a moment either, since nothing can exist for only a moment; thus the assumption is false.

The first thing to note about the argument is that it involves a dilemma. This is significant because it makes it clear that in order to show that shape is impossible one must rule out two possibilities, namely, that shape could exist over time and that shape could exist at a moment. Any argument against the reality of shape, therefore, is dependent upon some argument against the possibility of temporally enduring shapes, whether or not it appeals explicitly to diachronic considerations. This makes the diachronic part of the NPS argument of central importance to Leibniz's overall critique, since it is the only explicit diachronic argument to be found in Leibniz's writings. Fortunately, the argument is quite strong. In fact, it would very likely have been acceptable to a Cartesian. After all, Descartes would agree that motion in a plenum requires infinite (or, as he puts it, indefinite) divisions in matter, and he would also agree that in order for bodies to exist there must be motion. Leibniz simply draws the obvious conclusion from these premises, namely, that each and every part of the plenum must be in a continual state of modal alteration. (I discuss the justification for the first assumption in the next section.)

The synchronic aspect of the argument, however, is not so obviously sound. The central assumption of the argument is that "what exists only at a moment has no existence" (hereafter, the NME claim); and clearly, this claim is far from being obviously true. Nevertheless, I think Leibniz does have the resources to defend it. In fact, he offers an explicit justification in the text we are considering. The NME claim is true, he says, because what exists at a moment "starts and finishes at the same time." As Samuel Levey has pointed out, it may be that Leibniz is implicitly assuming that anything that exists must have a distinct beginning and end (Levey forthcoming (a)). If this is what Leibniz has in mind, then it would simply be contradictory to say that something exists only at a moment, since such a thing would not have a distinct beginning and end. Further argumentation would, of course, be needed to justify this implicit assumption, since without some justification the argument seems to beg the question quite egregiously; but it is certainly not a wildly implausible principle.

But even if we are skeptical about the soundness of his explicit justification, Leibniz could still justify the NME claim by appealing to his view on entities such as points, instants, lines, etc. According to Leibniz, such entities are not parts of things, nor are they entities out of which things could be composed. Rather, they are *extrema*, *termini* or limits of things (RB II, xiv, 10). As modes of (spatially or temporally) extended things, their existence is entirely dependent upon the extended things they bound. So, considered apart from that which they bound, they have no existence – they are mere abstractions. Of course, we might be able to conceptualize the end-point of a line segment or the first moment of a temporal interval; but apart from the extended things from which they were abstracted these points and instants are nothing more than mere mental entities. If Leibniz understands points and instants as mere abstractions, could he conceive of momentary states in the same way? There are two things that entities such as points, instants, and lines have in common: they are un-extended (at least along one dimension) and they can be conceived of as limiting something. For example, a line, which can be thought of as limiting an area of space, has no breadth; and an instant, which can be thought of as limiting a temporal sequence, has no temporal extension. If Leibniz thinks these entities are abstractions in virtue of these two features, then it seems he could quite easily assimilate momentary states to the same model. Momentary states, after all, are not temporally extended, and they can be conceived of as limiting an extended quantity such as a continuous flow of such states. Of course, there may be some relevant respect in which they are not analogous to limit entities such as points and instants. But it at least seems open to Leibniz to treat momentary states in the same way he treats other limit entities.

So far, I have been arguing that Leibniz has the resources to defend the NME claim. Other commentators, however, have been less convinced of its plausibility. Robert Adams, for example, has called it "a large and dubious assumption." But Adams' central worry is not that it is unmotivated but rather that it seems inconsistent with other things Leibniz either does or should hold. As Adams points out, "Leibniz himself must probably ascribe to monads perceptual states that do not endure, unchanged, beyond an instant" (Adams 1994: 231–2). And in fact, in his 1695 essay "A Specimen of Dynamics," Leibniz seems to say explicitly that forces or strivings exist at a moment: "there is nothing real in motion but a momentary something which must consist in a force striving [*nitente*] towards change" (GM VI 235; AG 118). Despite this apparent textual evidence, I am not convinced that Leibniz is committed to the existence of momentary states. For any textual reference to momentary states is consistent with an ontology in which such states are mere abstractions from the continuous flow of perceptions, forces, etc. Furthermore, I see no reason why Leibniz might need to ascribe such states to monads, nor do I see any reason that he should. If he were committed to a sort of temporal atomism with respect to states, he would need to have some account of the metaphysical individuation of those states (that is, some account of why something counts as one state as opposed to many); and I do not think he has such an account. But, of course, he would not need one if he simply denied the existence of such states.

If I am correct about Leibniz's views on momentary existence, he has the resources to defend the NPS argument; and since he does not express any worries about the argument, there is thus no real reason to doubt his commitment to its soundness. But this raises an important question: why, if he thinks the NPS argument is sound, does he develop an alternative synchronic argument that becomes dominant in his thinking about shape in the mid-1680s? After all, if he thinks the NPS argument is successful, why would he not simply rehearse this argument in his other discussions of shape from this period? Adams' interpretation would seem to provide a ready answer: Leibniz saw that the NME assumption is implausible or inconsistent with other metaphysical facts to which he is more firmly committed. But as ready as this answer may be, I think there is a more subtle explanation, one that is consistent with Leibniz's commitment to the soundness of the NPS argument: Leibniz eventually came to suspect that the Cartesians are committed to the existence of momentary states, and thus to the denial of the NME claim. Whether or not Leibniz's suspicions were correct, there is at least the suggestion of such a commitment to momentary states in Descartes' discussions of the divine preservation of created things. In the *Meditations*, for example, Descartes says that "the same power and action are needed to preserve anything at *each individual moment of its duration* as would be required to create that thing anew if it were not yet in existence" (CSM 2: 33, emphasis added). Assuming that Descartes' metaphysics is one of the central targets of Leibniz's attack, if Leibniz took such statements as indicative of a commitment to momentary states, the appeal to the NME premise would likely have struck him as a huge weakness in his argument. Of course, it is also consistent with my assessment of the NPS argument that Leibniz developed an alternative argument simply because he wanted to present as many good arguments as possible. But I think my explanation gets some support from the fact that later in the text in which the NPS argument is presented, Leibniz offers a different argument against the possibility of momentary shapes, one that is restricted to shapes in a Cartesian plenum. In the last paragraph of the text he says:

> In an instant, with motion not being considered, it is as if the mass were all united; and thus one can give it any shape as one wants. But also, all variety in bodies ceases; and, consequently, all bodies are destroyed. (A VI, iv, 1614; RA 299)

No shape could exist for only a moment, according to this argument, because shape is dependent upon motion and motion requires some period of time. Insofar as motion is necessary for the individuation and qualitative differences among bodies, as it is for a plenum physics of the sort the Cartesians endorsed, the material world at any moment is a completely homogenous mass and is thus without bodies – much less shapes.

This argument, which is similar to an argument Leibniz presents in a text from 1698 (G IV, 512–14; AG 164), is devastating to the possibility of momentary shapes, at least in a Cartesian plenum, and it does not appear to suffer from any of the problems Adams associates with the NME claim. And if Descartes is the primary target of the NPS argument, this is all he would need to rule out the synchronic horn of the dilemma. But this raises a further question, the answer to which reveals something important: why does Leibniz not simply appeal to this line of reasoning in ruling out the synchronic horn of the dilemma in the NPS argument? One possible answer is that Leibniz does not see a distinction between this line of reasoning and his justification for the NME premise, and so in stating the NME premise Leibniz means to be making the more limited claim that in a Cartesian plenum physics, nothing exists at a moment (or, at least, there is no individuation in the plenum). But I have strong doubts about this interpretation. Most importantly, the immediate justification given for the NME claim, namely, that at a moment a thing would start and finish at the same time, is quite general. In fact, it would seem to rule out the momentary existence of anything, whereas the latter argument is effective only against a Cartesian conception of body. I think it is much more likely that Leibniz appeals to the NME principle in the NPS argument because he wants an argument that rules out the existence of shape in any intelligible plenum conception of the world, not just the Cartesian conception, and he thinks the NME principle adequately accomplishes this. As we will see, the alternative synchronic argument that becomes dominant in Leibniz's thought in the mid-1680s has the same broad scope the NPS argument is intended to have. And this, I think, lends some support to the idea that Leibniz wants to rule out the existence of shapes in any type of plenum.

The Dominant Synchronic Argument

As we saw in the previous section, Leibniz has convincing reasons that there can be no shapes in a Cartesian metaphysics. And if I am right about the defensibility of the NME claim, the NPS argument shows something much stronger, namely that shape is impossible in any sort of plenum physics. Nevertheless, Leibniz develops an alternative synchronic argument that becomes dominant in his critique of shape. He offers a clear statement of this argument in a 1687 letter to Arnauld:

> Shape itself, which is of the essence of finite extended mass, is never exact and specific in nature, because of the actual division *ad infinitum* of the parts of matter. There is never a shape without inequalities, nor a straight line without curves intermingled, nor a curve of a certain finite nature unmixed with some other, and in small parts as well as large, with the result that shape, far from being constitutive of bodies, is not even a wholly real and specific quality outside of thought, and one will never be able to fix upon a certain precise surface in a body as one might be able to do if there were atoms. (G II 119; M 152)

Leibniz repeats the central argument of this passage at several places in the correspondence: the parts of body are actually divided to infinity; therefore, there is no fixed and precise shape (or surface) in body. And he presents a similar argument in a 1686 paper entitled "A Specimen of Discoveries About Marvelous Secrets":

> From the fact that no body is so small that it is not actually divided into parts which are excited by various motions, it follows that no determinate shape can be assigned to any body, nor is an exact straight line, nor a circle, nor any assignable figure of body found in the nature of things, though in the derivation of an infinite series certain rules are observed by nature. And so shape involves something imaginary, and no other sword can cut the knots we weave for ourselves by our imperfect understanding of the composition of the continuum. (A VI, iv, 312; P 81)

Diachronic considerations are clearly absent from these arguments. Leibniz is not concerned to point out that shapes are imaginary as a result of their always changing. Also absent are any references to the NME premise, the premise in the NPS argument that ruled out shapes at an instant. Instead, we find a synchronic argument in which the central premise is that the plenum is actually infinitely divided.

A slightly different formulation of the argument is found in "Primary Truths," an essay from around the same period. Here, the reason he gives for there being no determinate shape in actual things is that

> none can be appropriate for an infinite number of impressions. And so neither a circle, nor an ellipse, nor any other line we can define exists except in the intellect, nor do lines exist before they are drawn, nor parts before they are separated. Extension, motion, and

> bodies themselves . . . are not substances, but true phenomena like rainbows and parhelia. For there are no shapes in things. (C 522; AG 34)

Again, there is no suggestion in this text that Leibniz is appealing to diachronic considerations. But the argument is interesting in that Leibniz employs an additional premise that he does not mention in the other texts we have looked at so far, namely, that no shape "can be appropriate for an infinite number of impressions." Despite this difference, however, these versions of the argument have something very important in common: they both depend on the idea that determinate shape in body is precluded by the infinite complexity or infinite division of the plenum.

Before considering in more detail the relation Leibniz sees between shape and infinite division, there are two preliminary questions that must be addressed. First, in beginning the argument with the assumption that matter is infinitely divided, is Leibniz assuming from the outset that matter exists? And if he is, does this undermine my claim that these remarks are part of a general critique of realism about extended substance? Second, why does Leibniz think that matter must be actually infinitely divided in the first place?

The answer to the first question is that he is in fact assuming the existence of body and matter. But he is assuming this for the sake of *reductio*. Most of the statements of the shape arguments emerge in a dialectic in which Leibniz is trying to convince Arnauld that there is something incoherent about Cartesian extended substance. So it would be no surprise if Leibniz assumed the view that he is trying to show is incoherent, namely, that extended substance exists. We can answer the second question by considering what Leibniz thinks would have to be true if extended substance did exist. His thinking about this can be drawn from a variety of different contexts. Very briefly, it could be formulated as follows: if there is matter, it must be a plenum (that is, the world must be completely filled with matter in such a way that there can be no void); motion in a plenum requires infinite division; so, on the assumption that there is motion, matter must be infinitely divided.

Let us focus for a moment on the first two premises of this short argument. Leibniz has at least two reasons for thinking that the world must be completely full. The first is based on a principle that has been called the "Principle of Plenitude," which is in turn grounded in a more fundamental principle, the "Principle of Perfection." According to the Principle of Perfection, in his decision to create this world, God acted so as to maximize perfection or goodness. Perfection, in turn, "is nothing but quantity of essence." From this, Leibniz derives the Principle of Plenitude: "Out of the infinite combinations of possibles and possible series, that one exists through which the most essence or possibility is brought into existence" (G VII 303; LL 487). And the possible series that contains the most essence is one that is completely full (Rescher 1979: 27–30, 50–1; Rutherford 1995: 22–3). The second reason he believes the world is a plenum is that he rejects the existence of a vacuum in nature. One argument that he presents for this is based on the Principle of the Identity of Indiscernibles. Roughly, this principle says that two things cannot "resemble each other completely and differ in number alone [*solo numero*]" (G IV 433; AG 41–2). The argument, then, is that if there were a vacuum, there would be parts of space that differ only in number, which is impossible since it violates the principle.

The second premise is that motion in a plenum requires infinite division. Leibniz's central reason for believing this is that he agrees with Descartes' view that in a plenum all motion involves circuits and that in order for any parcel of matter to move through the irregular circuits that exist in the plenum, that parcel of matter must be infinitely divided into smaller parcels of matter. The reason infinite division is required could be summed up as follows: in order for a parcel of matter to pass through a narrow part of a circuit, it must alter its shape; and for this to happen it is necessary that each of its innumerable parts move with respect to its neighboring bodies; since bodies are individuated by their motions, any motion of a body is a real division of that body from the bodies that surround it; thus movement through irregular channels involves innumerable divisions (G IV 370; LL 393).

If we add one more premise to the two premises we have been considering, we get a formulation that reflects the general structure of Leibniz's reasoning about shape in the texts we have been considering in this section:

The infinite division argument

1 Assume: There is body/matter.
2 There is a plenum. (1)
3 Motion in a plenum requires infinite division. (premise)
4 So, matter is infinitely divided. (2), (3)
5 If matter is infinitely divided, then shape is an imaginary, non-objective property of things.
6 So, shape is an imaginary, non-objective property. (4), (5)

This formulation makes it clear that an account of the reasoning behind the conditional in step (5) is crucial to our understanding of the argument. Of course, the inference from infinite division to the impossibility of enduring shapes has already been explained in the NPS argument. But we still need an explanation of why infinite division makes momentary shapes impossible. Unfortunately, Leibniz is not very helpful on this point. In fact, in most presentations of the argument he says nothing about why we should accept it. He does, however, make one remark that I believe is intended to help clarify his thinking about the premise. Recall that in "Primary Truths" Leibniz appeals to a premise we do not find in any other text:

PT Principle: No shape can be appropriate for an infinite number of impressions.

Although this principle is somewhat obscure, it is at least the sort of claim that could be of some help in clarifying premise (5), for Leibniz very likely thinks infinite impressions are the result of infinite division. If this is right, then the conditional in (5) can be justified by the following argument: if there is infinite division, there are infinite impressions; and if there are infinite impressions, then there is no shape; thus, if there is infinite division, there is no shape. Of course, without an understanding of the PT Principle, we will be no better off than we were before, since the principle seems no less obscure than premise (5). It is thus extremely important that we look more carefully at what Leibniz might mean by the principle.

The first thing to note about the principle is that in the context in which Leibniz states it, the term "impression" has a physical rather than a perceptual sense. It might refer to a physical effect on something, such as a mark or dent on the surface of a body; or it might refer to that which brings about a physical effect. In either case, a natural way to understand the principle, and the way most commentators interpret it, is as claiming that no shape could have the degree of complexity it would have to have if it suffered infinite physical impressions. Put more simply: no shape could be infinitely dented or variegated. If this interpretation is correct, then the way Leibniz moves from the infinite division of matter to the lack of shape is via the PT Principle and the following premise: (7) if matter is infinitely divided, then if something has a shape, its shape must be infinitely complex. But what might be the relation between infinite division and shape (or lack thereof)? The idea would seem to be the following. Consider a body and its shape (either conceived or perceived). Because of the infinite division of the plenum, the body bounded by that shape is at any moment suffering the impressions of the motions of infinitely many surrounding bodies, and this would have to be reflected or expressed in the complexity of the shape. Furthermore, the same thing will be true for any part of the surface, since contiguous with any part of the surface there will be infinite bodies in motion relative to one another, bodies that are impressing themselves on that surface. And it is not just external bodies that impress themselves on the surface of the body; there are also motions of bodies within the bounds of the surface that would have an effect on the shape, since the body itself is infinitely divided. This is because, according to Leibniz, fluidity is the "fundamental condition" of the material continuum (RB II, xiii, 23), and so we must conceive of all bodies as composites of smaller parcels of matter that are in motion relative to one another, and we must think of each of those smaller parcels of matter as composites of yet smaller parcels of matter that are in motion relative to one another, and so on *ad infinitum*. The idea is thus that since any purported surface would suffer the impressions of an infinitude of internal and external bodies, the surface itself would have to be infinitely complex.

If this is the way Leibniz is thinking about the relation between infinite division and shape, then the remaining interpretive issue is to explain why is it impossible for a shape to be infinitely complex. There are two explanations that have been offered by commentators. The first, by Samuel Levey, is that in the early modern period something counts as a shape only if it can be described by traditional, finite geometry, and so an infinitely complex surface would, by definition, not be a shape. According to Levey, Leibniz's case against Descartes could be summed up as follows: an infinitely complex shape cannot be described by traditional geometry; if something cannot be described by traditional geometry, then that thing is not a shape; given the infinite impressions on the surface of any body, that surface could not be described in terms of traditional geometry; thus, Descartes is wrong: bodies have no shape (Levey forthcoming (b)). The second explanation, offered by Robert Adams, is simply that Leibniz would have thought that an infinitely complex shape is "an absurd and impossible monstrosity" (Adams 1994: 230). Adams does not provide any support for this claim, but it is possible he thinks there is some justification for it lurking in Leibniz's complex views about the infinite. For various reasons, I find both of these ways of understanding Leibniz's thinking about shape inadequate; furthermore, I do not believe Leibniz's discussions of the infinite provide sufficient resources to rule out the possibility of infinite shapes (Crockett 2004). Rather than discuss these interpretations here, however, I will instead lay out an alternative interpretation of the Infinite Division Argument that I believe makes better sense of the texts and makes a stronger case against the reality of shape.

An Alternative Interpretation

The central interpretive difficulty with the Infinite Division Argument is that it is hard to know how Leibniz understands the relation between the infinite division of the plenum and the impossibility of shape. The PT Principle, at least as it is commonly understood, suggests an interpretive direction, but it leaves us with a further puzzle that Leibniz never explicitly addresses, namely, the question of what justifies the principle. There is, however, an alternative way to understand the work this principle is doing that does not leave us with this question. On this interpretation, the upshot of the principle is not that given infinite impressions, any purported shape would have to be too complex to count as a shape or to be real. Rather, it is that given infinite impressions, there is nothing, metaphysically speaking, that we could characterize as the surface of a body. That is, any purported shape will at some level of analysis literally disappear, rather than merely reveal greater complexity than it did at the previous level. This is a subtle distinction, but I think it is significant for our understanding of Leibniz's critique of shape. The rest of this section will thus be devoted to explicating the distinction between these two ways of thinking about infinite impressions and shape.

To begin, it is worth noting that on both interpretations the PT Principle adds something new to the resources we have for understanding the move from the infinite division of the plenum to the impossibility of shape: the notion of physical impressions. In an infinitely divided plenum, any body that we consider (however it is individuated) is going to be impressed upon by infinitely many surrounding bodies. But there are different ways of conceptualizing this state of affairs, and these differences are important. On one way of conceptualizing matters, a way that I think is natural if we think the PT Principle is meant to rule out shapes of infinite complexity, we are to take for granted the intelligibility of a continuous, determinate surface, and then ask whether "it" could have the structure it would have to have in an infinitely divided plenum. We might conceive of this by thinking about the inside and the outside of a body as determinately separated by a metaphysical film, the integrity of which is taken as constant as we consider what would have to be true of it given infinite impressions. (Crudely, it might be helpful to imagine a balloon that is both filled with and buried in extremely fine sand.) This way of conceiving of a body in a plenum seems perfectly natural; and the result of thinking about the plenum in this way is that we are left with some conception of an extremely (in fact, infinitely) complex shape, and with the question of why such a shape could not exist.

One thing that makes this way of thinking about the complexity of a plenum so natural is that we have all had sense experiences in which we took a closer look at one part of the surface of some object, such as a billiard ball, only to find that the shape of that part was much more jagged or complex than it appeared with the naked eye. In such cases, our sensory examination did not, of course, reveal infinite complexity. But it did reveal substantially more complexity than was evident to the naked eye. And we can easily imagine that increasing the magnification on a part of the part we just examined would reveal that it is even more complex than it appeared. In fact, we can imagine that this would be true no matter how great the increase in magnification, or how many times we engage in this process of reexamination. It is not hard to see that this imaginative exercise assumes the "metaphysical film" conception of surfaces in a plenum. For no matter how detailed our analysis becomes we never lose track of the original surface. Certainly, at deeper levels of analysis the surface is going to be more complex than it appeared. But it is assumed that at each level we are noticing more detail about the same surface we considered at the previous level; and this is a way of assuming the determinate individuation of the body with which we started.

There is, however, another way we might conceptualize an infinitely divided plenum that does not assume determinate individuation at every level of analysis. According to this way of thinking about a surface in a plenum, no shape is appropriate for infinite impressions in the sense that being subject to infinite impressions undermines the metaphysical individuation of surfaces; shape is impossible because any purported shape will literally disappear at some level of analysis rather than simply revealing greater complexity. To see this, let us again start by thinking about a body and considering the fact that in an infinitely divided plenum, there are infinitely many bodies impressing themselves along its surface. But this time, rather than focus on a surface that we assume is metaphysically determinate and ask what would have to be true of "it" at any level of analysis, let us think more carefully about the very idea of a surface in an infinitely divided plenum. Let us again consider a billiard ball, and for the time being let us think about that object as abstracted from the plenum. We might grant that on closer analysis what we would find is greater and greater complexity of the apparent shape. But at some point in the analysis we would start to lose a conceptual grip on the surface we are considering. This is because a body in an infinitely divided plenum is really an aggregate of bodies that are individuated from one another by their motions. So eventually this picture of greater and greater complexity in "the shape" of the ball must give way to a picture of a surface as consisting of an aggregate of surfaces.

What follows from the fact that what appeared to be a relatively continuous surface is actually (at this level of analysis) an aggregate of surfaces? We might think it does nothing to undermine the object's having a continuous and determinate shape at all. After all, it seems we could simply draw a line along the exposed surfaces and take that as the shape. But, as I mentioned above, Leibniz thinks that fluidity is the fundamental condition of the plenum. So the (relatively solid) corpuscles that compose the macro-object (the billiard ball) cannot be perfectly hard, nor will they be perfectly contiguous with one another (i.e., perfectly packed together). Rather, they will themselves be aggregates of smaller bodies and will thus be separated from one another by some relatively fluid matter.

Given this micro-complexity, it becomes very difficult to answer the question: where is the shape of the body? One possible answer is that it is wherever there is an interface between matter and void; after all, we are at this point considering the "object" in abstraction from the plenum (say, in a vacuum). But this raises the further question of what it is we in fact abstracted from the plenum. At the time, we really did not pay much attention to the details of the abstraction; and now it seems crucially important. In saying, "Let us consider the billiard ball in abstraction from the rest of the plenum" did we mean, "Let us consider the corpuscles which compose the ball," or did we mean to abstract everything that is contained within some geometrical bound? There seem to be problems with either answer. If we meant the latter, then the abstraction had to involve some decision on our part about what the geometrical bound is, and how much complexity is to be accounted for in that geometrical bound. Given this, it is clear that we were responsible for delineating the shape, rather than the nature of reality. But if we meant the former, then even if there were some determinate set of corpuscles that constitute the body, the object would have no more a shape than a swarm of bees.

We can think about our abstraction on the following analogy. Imagine a glass jar filled with marbles. Now imagine that very spatial arrangement of marbles without the jar. This is in some ways an appropriate model for our billiard ball (as it is for each individual marble): the marbles are the corpuscles that compose the billiard ball, and the air is the relatively fluid matter that surrounds those corpuscles. Now in abstracting our billiard ball from the plenum, were we abstracting merely the corpuscles that compose the body (the marbles)? Or did we mean to also include the relatively fluid matter surrounding the corpuscles (the air around the marbles)? If we were abstracting only the corpuscles (or marbles), it seems clear that the aggregate has no determinate shape at all. And this becomes even more clear when we are careful to bear in mind that each corpuscle (or marble) is fluid too, and so is like the collection of corpuscles (or marbles) in all relevant respects. If, on the other hand, we were abstracting the corpuscles with the fluid matter, then there is a question about what determined the geometrical bounds (where the jar would be) in the first place. To a certain extent, this decision starts to look somewhat (though perhaps not entirely) arbitrary. One thing that is clear, though, is that in this case, we are the ones doing the carving: the boundaries are not metaphysically prescribed. And this only becomes more evident when we finally consider the body as part of the entire plenum, rather than abstracted from it, since the fluid matter that surrounds the corpuscles will be relatively continuous with the fluid matter that is moving between the corpuscles.

Finally, we must keep in mind that it will not help to say that what should be included as part of the body are the corpuscles plus whatever fluid matter is responsible for the corpuscles' bearing the relations they do to one another, and their having the properties they do individually. For in a plenum, there is a real sense in which every part of the plenum is at least partially causally responsible for the states of, and relations among, every other part. Of course, this might show that the sort of abstraction we considered is deeply unintelligible. But even if it is, this does not undermine the point that infinite complexity in the plenum entails that surfaces are illusory rather than real properties of things.

On this interpretation of Leibniz's reasoning, therefore, we are to see that the infinite division of the plenum precludes the existence of shape by focusing on the possibility of individuating surfaces in a plenum physics, not by focusing on the amount of complexity it is possible for a determinate surface to express. In other words, the problem caused by infinite impressions is not that they result in an infinitely messy surface; rather, it is that they make metaphysically individuated surfaces impossible. Of course, on both readings any particular shape is shown to be an abstraction from the infinite complexity of the plenum. But on this alternative interpretation, it is not an abstraction from a surface that is actually more complex than it seems. Rather, it is an abstraction from something that has no real existence. This is the sense, then, in which no shape is appropriate for infinite impressions: there are no surfaces that could do the work of demarcating or individuating bodies in a plenum in which every parcel of matter is being impressed upon by the infinite bodies that compose that body and the infinite bodies that surround it. Put more simply, no infinitely divided plenum, including the Cartesian variety, has the resources to make sense of the real individuation of bodies from one another.

Shape and Idealism

Leibniz's arguments against the reality of shape are clearly intended to undermine the intelligibility of a Cartesian plenum theory; and to this extent I think the arguments are successful. Furthermore, I think the arguments are sufficient to show that shape cannot exist in any type of plenum theory. But do they show anything stronger? That is, do they undermine the intelligibility of realism about matter more generally? I do not think they do, at least on their own. Certainly, showing that shape is impossible is not sufficient for establishing immaterialism. However, if these considerations about shape are conjoined with other assumptions, assumptions which are clearly Leibnizian, we do get something that begins to resemble an argument for immaterialism. In this section, I shall sketch the way such an argument would go.

In section XII of the "Discourse," Leibniz draws an explicit conclusion from his arguments against the reality of the modes of extension: shape (and the other Cartesian modes) "cannot constitute any substance" (AG 44). This clearly undermines the Cartesian conception of extended substance, since Cartesian bodies are nothing more than extension in length, breadth, and depth. But the arguments against the possibility of shape, at least as I interpret them, also get us a slightly stronger conclusion, namely, that shape cannot be a real property of anything. For the arguments against the reality of shape depend on assumptions to which Leibniz is firmly committed: if there is matter it is a plenum, and if there is a plenum it is actually infinitely divided. And if shape cannot be a genuine property of anything, it is certainly true that it cannot be a property of a substance. This, of course, does not show there can be no extended continuum, since we can conceive of an extended continuum in which there are bodies that are not determinately individuated from one another, or that have fuzzy, indeterminate shapes. But there is a central Leibnizian thesis that seems to raise problems for this conception of the plenum: the aggregate thesis.

According to the aggregate thesis, beings by aggregation are only as real as the entities out of which they are aggregated. In other words, the reality an aggregate has is a function of the reality of the ultimate constituents out of which it is aggregated. Leibniz appeals to this principle at many points throughout his career. One of the best statements of it can be found in a letter to Arnauld: "entities made up by aggregation have only as much reality as exists in their constituent parts" (G II, 72; M 88). A direct implication of this thesis is that if an aggregate is real or substantial, the elements which constitute the aggregate must be real or substantial; that is, they must satisfy the conditions on being a substance. With this thesis in hand, it is possible to formulate a *reductio* of any genuinely (mind-independently) extended continuum:

1 Assume: There is a metaphysically real, extended continuum (i.e., a plenum of matter, however constituted).
2 There must be some account of the composition of any real continuum.

This is a principle that Leibniz takes to be fundamental (G II 98, 119; M 123, 153).

3 If a continuum is composed, it is an aggregate.
4 The reality of an aggregate depends on the elements out of which it is aggregated. (Aggregate Thesis)
5 Something that is genuinely extended cannot be composed of unextended entities.

Although early in his career Leibniz flirted with the idea that a continuum could be composed of extensionless points, he had given up this idea by the period in which we find the arguments against the reality of shape.

6 So the elements of the continuum must be extended. (1)–(5)
7 And the elements must not be aggregates; they must be simple and substantial. (1)–(4)
8 So there must be extended substances. (6), (7)
9 If there are extended substances, these entities must be metaphysically individuated from one another in the plenum; that is, there must be a fact of the matter about where one substance ends and another begins.

According to Leibniz, substances are completely determinate, self-contained, and independent of every other substance. It would seem to follow from this that there would need to be absolutely determinate boundaries or surfaces in virtue of which extended substances are individuated from one another.

10 But there is no fact of the matter about the individuation of extended things in a plenum. (The arguments against the reality of shape)

So, on the assumption that there is genuine, mind-independent extension, the aggregate thesis demands there be extended substances that ground its reality. But extended substances would need to be metaphysically individuated from one another, since it could not share a part with any other substance. Thus, the assumption (that an extended continuum exists) is false.

If this argument is sound, then Leibniz's arguments against the reality of shape have at least the potential to play a central role in an argument for immaterialism. And I think it is likely that he did see the connections between the ideality of the modes of extension and impossibility of genuinely (mind-independently) extended substance.

But if he was aware of the strength of his arguments against the modes of extension, a puzzle arises about how we are supposed to understand a second conclusion that he mentions in section XII of the "Discourse." After claiming that the modes of extension are imaginary, he goes on to say that "we must necessarily recognize in body something related to souls, something we commonly call substantial form" (G IV, 436; AG 44). This sounds as if Leibniz is suggesting we need to add something to the Cartesian theory of matter to get an adequate account of body, rather than that we should conclude that there is no matter at all. As paradoxical as it might sound, I believe the truth lies somewhere in between these two options. That is, I think the correct interpretation is that there is a sense in which Leibniz wants to deny the reality of matter and a sense in which he wants to show that something needs to be added to the Cartesian conception to render natural philosophy intelligible.

There are two ways of thinking about what it is that needs to be "recognized in body." In the "Discourse" Leibniz says that we need to recognize something related to souls or substantial forms, yet it is clear from other passages in the "Discourse" that he is concerned to show that an intelligible physics must be one in which dynamical forces play a central role. In the end, I think there is nothing inconsistent in thinking that Leibniz wants to make both of these points. But this depends on a careful interpretation of what Leibniz is arguing for.

Let us begin by thinking about the conclusion that we need to recognize something like soul or substantial form in body. This could mean at least two different things. Commentators who read Leibniz as having an immaterialist metaphysics in this period think he is claiming, roughly, that what we need to recognize is that body is the phenomenal result of an aggregate of simple, immaterial substances, what he calls "monads." This is the sense in which souls or forms are "in" body. However, other commentators have argued that, at least in this period, form is to be understood as one aspect of a hylomorphic composite of form and matter. So what we need to recognize is that without form, the material continuum is completely homogenous, with no individuation or identity over time of its parts, but with form, the idea of a genuinely extended, material continuum is perfectly intelligible. A thorough assessment of this general line of interpretation is not possible here. But I think a brief comment about it is in order. The leading proponent of this view, Daniel Garber, seems to think that Leibniz's arguments against the reality of shape are intended to show only that shapes are not real unless there are "quasi-Aristotelian" substantial forms; after forms have been added to extension there is no problem with completely determinate shapes in the plenum (Garber 1985). In fact, according to Garber, form plays the critical role of marking out metaphysical distinctions in matter; it plays the role of a metaphysical cookie-cutter (albeit one that is always changing its shape). But how can we square this with Leibniz's arguments against the reality of shape? As I have already mentioned, the basic premises of the arguments are claims to which Leibniz remains firmly committed throughout his career. So if Garber is right, there must be some way of understanding the conclusion that would make it consistent with the cookie-cutter account of form. But if there are no metaphysically determinate shapes, it would seem there could be no fact of the matter about where one body ends and another begins.

In the "Discourse," it is clear that Leibniz also wants to point out that force, a physical or dynamical property, must be appealed to in any intelligible physics. Is it possible that this is what he means when he says in Discourse XII that we need to recognize something akin to substantial forms in body? I think it is unlikely. To be sure, Leibniz thinks that a central problem with Cartesian physics is that it does not appeal to forces, and as a result gets the rules of motion and impact wrong. But this is a point about physics, not ground-floor metaphysics. And having a well-developed physics, as Leibniz does, is perfectly consistent with having an immaterialist ontology. In fact, one of the points of showing the Cartesians that the true physics requires forces is that it also shows them that their metaphysics of matter is too austere to account for the dynamical properties of bodies, since physical forces must be grounded in something metaphysical (G IV, 444; AG 51).

Let us consider, nevertheless, whether force is the sort of property that could turn homogenous, unindividuated matter into something real. This way of putting it, of course, is not ideal. For, as Robert Sleigh says, "No one is likely to think that with the injection of force or form into shapeless mass, it literally 'shapes up'" (Sleigh 1990: 112). But there might be a different way of construing matters. In his forthcoming analysis of the shape texts, Samuel Levey offers a different way of stating the point about the relation between shape and force. According to Levey, force is the source of particular shapes in things:

> What it is for there to be matter arranged in a certain way is for there to be a certain nexus of forces acting in a given space . . . Extension itself is to be understood as a diffusion of resistance, and thus the specific diffusion of it into a given space constitutes a body of a given shape. (Levey forthcoming (b))

This may well be the best way of filling out a realist ontology on Leibniz's behalf. Nevertheless, I do not find it convincing as an interpretation. I think it is doubtful that the texts Levey cites in support of it are meant to spell out Leibniz's fundamental metaphysics. In fact, I think it is much more likely that Leibniz's claim that force "fixes shapes in matter" (G IV, 397), or his claim that extension is diffused resistance, is meant to be a piece of physics, rather than a piece of ground-floor metaphysics. Furthermore, Leibniz states explicitly in a text dated from this period that forces are reducible to the perceptual properties of simple substances: "Substances have metaphysical matter or passive power insofar as they express something confusedly, active, insofar as they express it distinctly" (G VII, 322; LL 365). (For Leibniz, "expression" is a technical term; but when it is applied to substances, such as minds, the term is (very roughly) synonymous with "perception.") This text, then, suggests that any talk of the diffusion of forces and the fixing of shape is to be understood as applying at the level of physics, rather than the level of deepest metaphysics.

In the end, there is a sense in which Leibniz wants to deny the reality of matter and a sense in which he wants to show that something needs to be added to the Cartesian ontology to render natural philosophy intelligible. His ultimate view is that the world as it is in itself consists solely in unextended, active, simple substances that bear neither causal nor spatial relations to one another. Nevertheless, he also believes physics is central to our understanding of the natural world, and that it is therefore important that we have the explanatory resources to make the phenomena intelligible. What Cartesian physics lacks, he argues, is a notion of physical force. But he does not think that we can retain a realist conception of body by simply adding a notion of force to Descartes' geometrical conception of matter as extension. Certainly, we can offer explanations of natural phenomena that are sufficient in physics by appealing to the properties of extension and dynamical properties. But there must be a deeper story about what grounds, metaphysically speaking, the dynamical properties of bodies that are essential to corporeal phenomena. And this is the story of the Monadology (G VI 607–23; AG 213–25). Leibniz's ultimate metaphysics is thus immaterialist, rather than realist. Nevertheless, there is an important sense, for Leibniz, in which matter is real. Our perceptions of bodies are not like mere dreams, states of mind that do not represent real things outside the mind. Rather, our perceptions of a world outside the mind represent a real world – a world of monadic reality. To this extent, then, bodies are real. But this only makes sense if we see them as "well-founded" in monadic reality; and this is what Leibniz means when he says we need to recognize something like souls or substantial forms in body.

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